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CALCULATION OF THE BASE PRESSURE IN TWO-DIMENSIONAL SUPERSONIC FLOWS

L. V. Gogish and G. Yu. Stepanov[1]

Description of a general method for determining the base pressure in complex two-dimensional supersonic flows. A two-dimensional isentropic supersonic flow is calculated in approximation, by the method of linear characteristics using a given separation-flow hodograph. Experimental and calculated results for the base pressure on a cylinder end, on the end bases of a plane channel, and in a nozzle are in fair agreement. An analysis of the dependence of the base pressure on the characteristics of a separation flow is conducted, using an engineering formula.

In the general literature concerning the determination of base pressure in supersonic flows, particular attention is given to investigating the behavior of the dissipative layer on the boundary of the separation region. The effect of the properties of the external inviscid flow, sometimes depending on changes in the magnitude of base pressure by several degrees, is investigated to a lesser extent. Then an approximation method for calculating base pressure in various types of real supersonic flows is presented.

1. **Formulation of the problem.** We shall assume in our discussion that: a: the variation in parameters in the external supersonic flow is isentropic and b: the effect of the initial thickness of the boundary layer may be disregarded. The external isentropic flow in the base region is broken down as usual into: (1) undisturbed flow, (2) turning flow at the edge, (3) detached flow in the base region and (4) compressive flow in the combination region. /109*

As was first shown by Chapman and Korst (refs. 1 and 2), the behavior of the dissipative layer in the region of separation is determined chiefly by the localized parameters of this layer associated with the selected self-similar velocity profile and the permissible degree of compression in the region of flow combination. Since the degree of compression in the combination region is determined by the local joining angle of the external inviscid flow behind the separation region, it is convenient to use the permissible angle of displacement of the dissipative flow at the point of attachment θ° (fig. 1) as the basic parameter which characterizes the viscous layer. The base pressure will then be determined from the condition of coincidence between the permissible angle of turn of the viscous layer and the gas-dynamic angle formed by the stream line of the external inviscid flow at the point of attachment of the detached flow.

Let $\varphi=\lambda_i/\lambda$ be the relative velocity on the stream line of constant flow rate in the viscous layer. The permissible degree of compression in the combination region is determined by the condition of penetration of the stream line of constant

*Numbers given in margin indicate pagination in original foreign text.

[1]Moscow

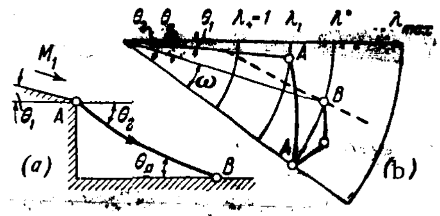

Figure 1. Representation of the detached flow in the physical plane (a) and in the plane of the hodograph (b), and broken line indicates behavior of permissible angle of turn of the dissociative layer at the point of attachment of the detached flow.

flow into the wake region (refs. 1 and 2) $p_{i*}=p'$ which, assuming isentropic compression in the wake, gives

$$\frac{\pi(\lambda')}{\pi(\lambda^\circ)} = \frac{1}{\pi(\varphi\lambda^\circ)}, \quad \lambda = \frac{w}{a_*}, \quad \pi(\lambda) = \frac{p}{p_*} = \left(1 - \frac{k-1}{k+1}\lambda^2\right)^{\frac{k}{k-1}} \tag{1.1}$$

Here w is the flow velocity, a_* is the critical velocity, p is the static pressure in the flow, p_* is the overall pressure; the superscripts $^\circ$ and $'$ refer to parameters on the boundary stream line of the external inviscid flow in the base region and in the wake region respectively. If it is assumed that the shock of closure is weak, $p^\circ_* \approx p'_*$, then the permissible angle of turn of the dissipative layer in the combination region θ° will be equal to

$$\theta^\circ = \nu(\lambda^\circ) - \nu(\lambda')$$

$$\nu(\lambda) = \text{[illegible]} \left\{ \text{[illegible]} \frac{\lambda^2 - 1}{1 - k'\lambda^2} \right)^{1/2} \text{[illegible]} \left(\frac{\lambda^2 - 1}{1 - k'\lambda^2} \right)^{1/2} \text{[illegible]}, \quad k' = \frac{k-1}{k+1} \tag{1.2}$$

Here $\nu(\lambda)$ is the Prandtl-Meyer function.

In figure 2, a is the function $\varphi(M)$ calculated for three self-similar velocity profiles in the turbulent boundary layer (assuming constant overall flow temperature); curves 1, 2 and 3 correspond to the relationships /110

$$\varphi = \text{[illegible]} (1 - \eta^{1/\text{[illegible]}}) \text{[illegible]} = \text{[illegible]}$$

where η is the relative transverse dimension of the layer, while b in figure 2 gives the corresponding function $\theta^\circ(M)$ calculated from formulas (1.1) and (1.2). The points given in figure 2b are obtained on the basis of various experimental data for base pressure on the face of flat and annular projections in a homogeneous supersonic flow. The relationship $\theta^\circ(\lambda)$ represented in figure 1b in the plane of the hodograph (for k=1.4) is satisfactorily approximated by a straight line in the range $1.1<\lambda<2.1$.

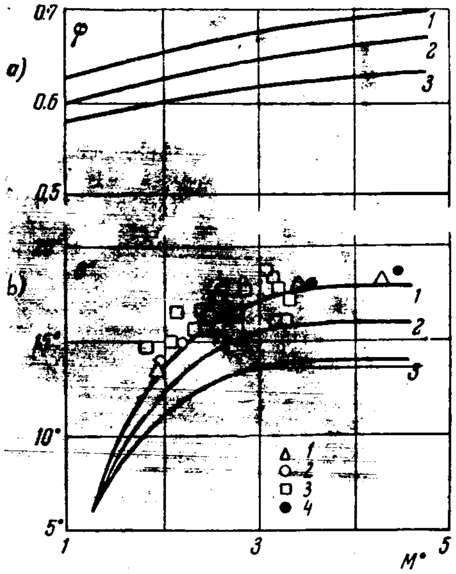

Figure 2. (a) Relative velocity on the stream line of constant flow rate in the turbulent boundary layer of a jet as a function of the reduced velocity of the external flow. (b) Permissible angle of turn of a turbulent jet layer as a function of the Mach number of the external flow; experimental points: 1, reference 1; 2, reference 7; 3, data of A. N. Timoshin; 4, data of the authors.

We note that the function $\theta^\circ(M^\circ)$ which speaking generally may apparently be considered universal depends both on the Mach number M and on the temperature factor for the viscous layer Θ and the relative density of injection into the base region

$$Q = (\rho w)_2 / (\rho w)_1 \, l / h$$

Here l is the length of the boundary stream line in the separation region, h is the height of the end plane.

Thus we obtain $\theta^\circ = \theta^\circ(M^\circ, Q, \Theta)$.

The permissible angle of displacement may be determined both by purely experimental methods and by calculation, e.g., using the Korst-Chapman theory.

Let us designate the total change in the angle of inclination of the boundary of inviscid flow in the entire separation region by $\omega = \theta_2 - \theta^\circ$; θ_1 is the angle of inclination of the wall in front of the end plane. Then, according to the flow diagram given in figure 1a, we obtain

$$\nu(M^\circ) - \nu(M_1) + \theta_1 = \theta^\circ(M^\circ) + \omega \qquad (1.3)$$

Equation (1.3) determines the relative base pressure $\pi(M^\circ)$ in a two-dimensional supersonic stream, assuming that the function $\theta^\circ(M^\circ)$ is universal and that $\omega(M_1, M^\circ)$ is determined by an additional construction. The function $\omega(M_1, M^\circ)$ is determined by constructing the inviscid flow in the separation region. The method of linear characteristics (ref. 3) should be used as the simplest means for solving the problem.

In analyzing the effect which the quantities θ_1 and ω have on the magnitude of the base pressure, it is convenient to use the diagram of the epicycloids for plane flow (fig. 1b) with the superposed behavior of the permissible angle of turn of the viscous layer according to figure 2b (curve 1). We note that the diagram of the epicycloids in figure 1b may be used for a direct solution of equation (1.3) in the plane of the hodograph in the case of plane detached flow, which substantially simplifies subsequent construction of the flow in the physical plane. /111

When M_1= const and ω=0, the relative base pressure p°/p_1 increases with an increase in θ_1 reaching unity at $\theta_1=\theta^\circ$. With a further increase in the angle θ_1 the relative base pressure becomes greater than unity and a weak compression shock develops at the edge of the end plane. The intensity of this shock, and consequently the degree of increase in the base pressure, have an upper limit determined by the critical pressure ratio (ref. 4) maintained by the boundary layer in front of the end plane. When there is no boundary layer, for instance when separation takes place at the leading edge of a body, this limitation is replaced by the condition of existence of a supersonic flow in the neighborhood of the detached flow. When there is a forward or edge shock in front of the separation region, the reduction in the overall pressure in this shock should be taken into account.

When M_1= const and θ_1=0, the relative base pressure p°/p_1 increases with an increase in ω reaching unity at $\omega=-\theta^\circ$. Similar flow develops in the case where a compression shock is incident on the boundary of the separation region (which, in particular, introduces an error into the results of some experimental studies of base pressure). An important characteristic in this case is the stepwise change in base pressure and in the entire configuration of the detached flow with a slight displacement of this shock through the transition point M=1 in the wake.

The validity of formula (1.3) is confirmed below by comparing the results of calculations with experimental data based on examples of two-dimensional detached flows.

2. Determination of the base pressure on the end of a cylinder in an infinite supersonic streamline flow. In conformity with the method of linear characteristics (ref. 3) we shall base our construction of the contour of the inviscid jet behind a cylinder (fig. 3a) on the analogous plane flow with constant parameters along the last characteristic of the expansion fan OA.

Using the equation of continuity for the axisymmetric flow along the first-approximation linear characteristics of the first and second families of OA and AB, we obtain a simple formula for determining the contour of the jet

$$r_B = \frac{1}{n}\left[1 + (n-1)r_A^2\right] \left(n = \frac{\sin(\theta^\circ + \alpha^\circ)}{\sin(\theta^\circ - \alpha^\circ)}\right). \qquad (2.1)$$

Here r_0 is the radius of the end of the cylinder, which is taken as equal to unity.

As shown in reference 3, the results of calculations by approximate formulas agree satisfactorily with the exact solution (ref. 5) obtained by the method of characteristics using a computer. Therefore the localized angle of inclination of the jet boundary to the axis as a function of the distance to the axis of symmetry of r is given by the formula

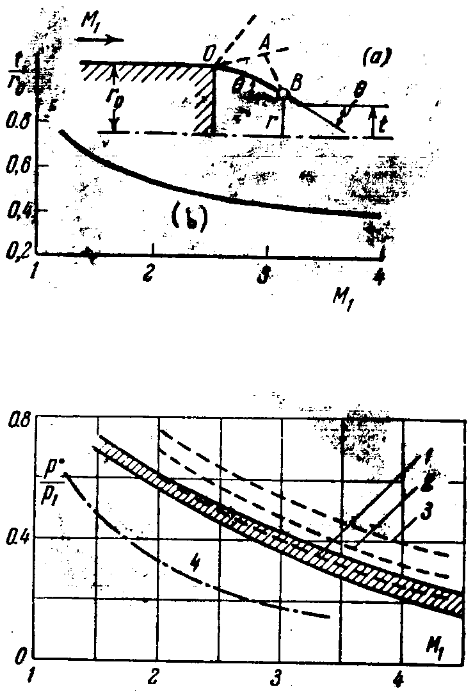

Figure 3. Base pressure on the end of a cylinder as a function of the Mach number of the external flow: shaded region--generalized experimental data from reference 6. 1, 2, 3, theoretical curves; 4, base pressure behind a projection in a plane flow (ref. 1).

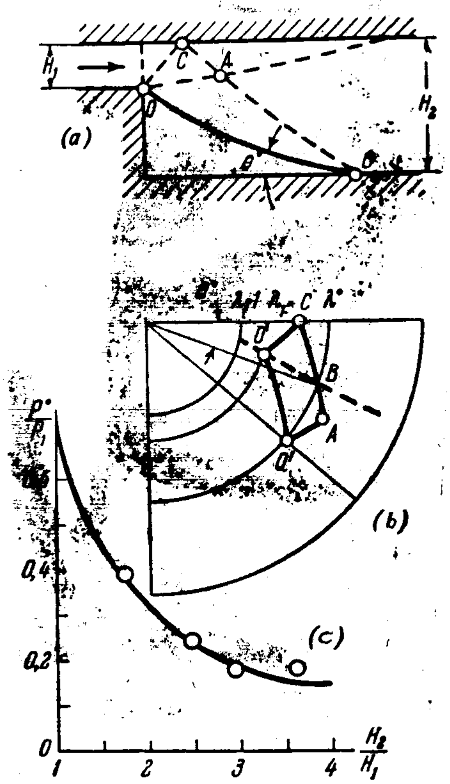

Figure 4. Base pressure in a plane supersonic flow in a channel with instantaneous expansion. Representation of the flow in the physical plane (a) and in the plane of the hodograph (b).

$$\operatorname{tg}\theta = \sin(\theta^\circ - \alpha^\circ)\left[\cos(\theta^\circ - \alpha^\circ) - \frac{\sin 2\alpha^\circ}{\sin(\theta^\circ + \alpha^\circ)}\,\frac{n}{n-1}\sqrt{\frac{r^2(n-1)}{r^2 n - 1}}\right]^{-1} \tag{2.2}$$

The magnitude of the base pressure is determined by the condition of coincidence between the localized gasdynamic angle of inclination of the contour for the inviscid jet and the permissible angle of the dissipative layer $\theta^\circ(M^\circ)$. The finite thickness of the viscous wake t/r_0 should be taken into account in this case.

Taking the relationship between t/r_0 and the Mach number M_1 of the oncoming flow from the experimental data of reference 5 (fig. 3b), we may easily determine the relationship between the numbers M_1 and M° and correspondingly the relative base pressure p°/p_1 as a function of the number M_1.

The results of calculations of this type, which were done for three velocity profiles in the jet, are compared in figure 3d, with the generalized experimental relationship for the base pressure on the end of a cylinder taken from reference 6. As may be seen from figure 3d, the results of calculations for the profile $\varphi=\frac{1}{2}(1+\mathrm{erf}\ \eta)$ correspond to the experimental data of all Mach numbers M as satisfactorily as for the case of plane flow.

3. Determination of the base pressure developed in a supersonic plane flow in a channel with instantaneous expansion. Figure 4 shows the gasdynamic diagram for flow of a supersonic stream of ideal gas (with a Mach number of M=1 in a narrow cross section) in a plane channel with instantaneous expansion, in the physical plane (fig. 4a), and in the plane of the hodograph (fig. 4b). The base pressure is conveniently calculated by assuming that the base pressure or the number M^{o} is given and then determining the value of the transverse dimension H_2/H_1 from the known hodograph. For this purpose it is necessary to find consecutively the position of points C, A and B in the physical plane using the characteristic directions. Segments of the characteristics may be constructed from the average parameters on the ends of the segments (with regard to the fact that there are two sheets for the representation of the flow in the plane of the hodograph). The position of the point of attachment B may be determined in two ways: from the intersection of the characteristic AB and the average linear direction of OB, and from the length of the segment of the characteristic AB, which was determined by using the equation of continuity. The results of the calculations, which are shown by the solid curve in figure 4c, coincide satisfactorily with the experimental data of reference 1 (experimental points).

Let us point out that this inverse problem of determining the characteristic geometric dimensions of detached flow in the physical plane from the known hodograph is extremely simple since in this case it is sufficient to construct only the individual characteristics which bound the region of influence. Solution of the direct problem is associated with calculating series of jet contours and selecting the one which intersects the wall at the permissible angle $\theta^{o}(M^{o})$.

A trial-and-error solution is required for calculation of analogous axisymmetric flow, which may also be constructed by the method of linear characteristics.

4. Base pressure on the end of a plane Prandtl-Meyer nozzle as a function of the available pressure ratio.[1] The effect which the wave structure of an external isentropic flow and its boundaries have on the base pressure may be conveniently analyzed on the basis of the example of flow in a plane nozzle with a single angular point shaped along the stream line of Prandtl-Meyer flow (fig. 5). At a given pressure of the external medium p_1, expansion of the flow around the angular point A takes place in a simple wave with an intensity determined by the available pressure ratio $1/\pi=p_*/p_1$, while the Mach number M of the flow on the extreme characteristic of the expansion fan is equal to M_1. Research shows that /113

with the flow around the curved surface of the nozzle section located downstream, a simple compression wave is developed which is reflected from the free boundary of the jet in the form of a simple expansion wave.

1 This section of the work was done with the participation of T. S. Soboleva.

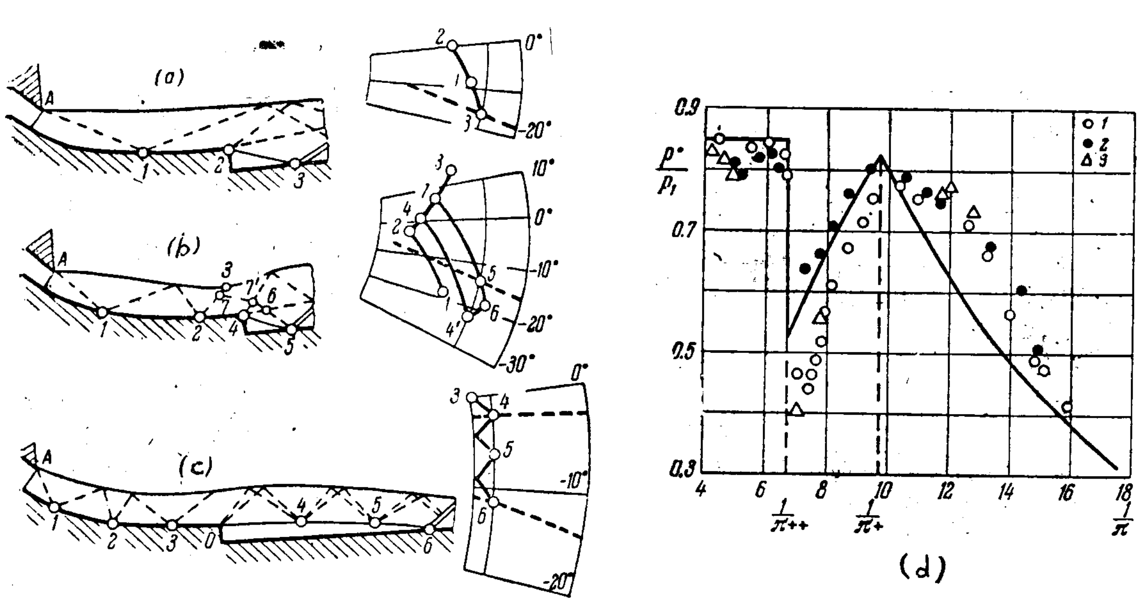

Figure 5. Relative base pressure on the end of a Prandtl-Meyer nozzle as a function of the available pressure ratio. Representation of the flow in the physical plane and in the plane of the hodograph at (a) $1/\pi_*<1/\pi<1/\pi_0$; (b) $1/\pi_{**}<1/\pi<1/\pi_*$; (c) $1/\pi<1/\pi_{**}$.

With flow around the angular point 0 located on the end, a simple expansion wave develops which interacts with the waves already in the stream and with the free boundary of the jet to determine the magnitude of the base pressure.

With an increase in the given quantity $1/\pi$ from the value $1/\pi_0$ (when the stream line in the angular point A is parallel to the x-axis, the Mach number M is equal to M_0) there are three distinct flow conditions in the base region which may be separated by the characteristic values $1/\pi_*$ and $1/\pi_{**}$. When $1/\pi_*<1/\pi<1/\pi_0$ the expansion wave reflected from the free stream line does not interact with the separation region (fig. 5a). The base pressure in this case is determined immediately by formula (1.3) when $\omega=0$

$$\nu(M^\circ) - \nu(M_1) + \theta_1 = \theta^\circ(M^\circ), \quad \theta_1 = \nu(M_0) - \nu(M_1) \tag{4.1}$$

When $1/\pi_{**}<1/\pi<1/\pi_*$ (fig. 5b), the reflected expansion wave interacts with the separation region. The intensity of this interaction increases with a reduction in $1/\pi$. Calculation of the base pressure in this case is done by trial and error, using the epicycloid diagram as in section 3. The given value of M° and the known flow hodograph are used to determine the height of the end by consecutive construction in the physical plane of points 2, 7, 7', 6 and finally the point of attachment 5. When $1/\pi<1/\pi_{**}$ (fig. 5c), a nearly homogeneous supersonic jet flows around the end with a wave structure following that in the separation region which is determined chiefly by the intensity of a simple wave developed during flow around the end. The magnitude of the base pressure in this case is determined by formula (1.3) where $\omega=\delta_1+\ldots+\delta_n$ (δ is the angle of turn of the flow in the wave, n is the number of incident and reflected waves on the boundary of the separation zone). In particular, if we are given a supersonic homogeneous flow of limited width with the number M_1 in the cross section before the end, while the number $M=M^\circ$ on the boundary of the separation zone, then $\delta-\nu(M^\circ)-\nu(M_1)$ and $\omega=(n-1)\delta$; and formula (1.3) is transformed to

$$\nu(M^\circ)-\nu(M_1)=n^{-1}(\theta^\circ-\theta_1) \tag{4.2}$$

When n=1, formula (4.2) determines the base pressure on the flat end of a body in an infinite streamline flow. When $n\neq 1$, the base pressure changes sharply in comparison with the case n=1, together with the effect of the initial angle of inclination θ_1. Even in this extremely simple case the number of waves n depends on three geometric and gasdynamic parameters--the transverse dimension of the jet, and the numbers M_1 and M° -- and is determined by construction of the flow in the physical plane. Generally speaking, the solution may be indeterminate since it is possible for flow around the end to take place with various numbers of simple waves in the jet.

Figure 5g shows the results of measurement of the relative base pressure p°/p_1 from the available pressure ratio $1/\pi$ in a Prandtl-Meyer nozzle with $\theta_*=42^\circ$ and $h^\circ/h_*=0.83$ (θ_* is the angle of inclination of the stream in the critical cross section, h° is the height of the end and h_* is the width of the critical cross section). The same figure gives the results of calculations made by the method outlined above, which agree satisfactorily with the experimental data. The discrepancy between the approximate calculation and the experimental data is explained by some displacement in the angular point of the nozzle A during the experiment, and also, apparently, by errors in calculation of the external flow associated with schematization of the supersonic flow and disregard of the viscous wake on its outer boundary.

Let us point out that the first characteristic value of the pressure ratio $1/\pi_*$ which determines the continuous flow condition diagram may be found theoretically. However, calculation of the quantity $1/\pi_{**}$ is associated with time-consuming constructions of the flow in the region of the wake with regard to the interaction of compression shocks with each other and with the wake, so that a reliable determination of this quantity $1/\pi_{**}$ is presently inconvenient.

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